

Magnetic instability due to flux avalanches in superconductors

N. A. Taylanov and M. Samadov

National University of Uzbekistan, Uzbekistan

(Dated: December 16, 2014)

Abstract

We have studied theoretically the space-time evolution of the thermal and electromagnetic perturbation in a superconductor with the linear current-voltage characteristic in the flux flow regime. On the basis of a linear analysis of a set of differential equations describing small perturbations of temperature and magnetic field we will found that under some conditions an instability may occur in the sample, which takes into account an inertial properties of the vortices mass.

Key words: nonlinear equations, oscillation, vortex mass, critical state, flux flow.

Bean's critical state with its spatially nonuniform flux distribution is not at equilibrium and under certain conditions the smooth flux penetration process becomes unstable [1-5]. The spatial and temporal development of this instability depends on the sample geometry, temperature, external magnetic field, its rate of change and orientation, initial and boundary conditions, etc. Instabilities in the critical state result in flux redistribution towards the equilibrium state and are accompanied by a significant heat release, which often leads to the superconductor-to-normal-transition. The basic instability observed in Bean's critical state is the flux jump instability, which was discovered already in the early experiments on superconductors with strong pinning [4].

Let us assume that a small perturbation of temperature or flux occurring in Bean's critical state. This perturbation can be caused by an external reason or a spontaneous fluctuation arising in the system itself. The initial perturbation redistributes the magnetic flux inside the sample. This flux motion by itself induces an electric field which leads to dissipation. This additional dissipation results in an extra heating which in turn leads to an additional flux motion. This "loop" establishes a positive feedback driving the system towards the equilibrium state. The flux jump instability exhibits itself as suddenly appearing flux avalanche and Joule heat release [4, 5].

Recently, Chabanenko et al. [6] have reported an interesting phenomenon in their experiments - convergent oscillations of the magnetic flux arising from flux jump avalanches. The authors argued that the observed oscillations due to flux avalanches can be interpreted as a result of the existence of a definite value of the effective vortex mass. Thus, it is necessary to take into account collective modes, i.e., the inertial properties of the vortices in studying the dynamics of the flux avalanches. Prior to the jump, the mixed state of superconductors is characterized by nonuniformly distributed magnetic induction localized near the surface. As a result of the avalanche, the flux rushes from either sides of the sample towards the center [6]. Two fronts of the penetrating flux collide in the center of the sample and, owing to the existing vortex mass, give rise to the local surplus density of the magnetic flux that exceeds the value of the external magnetic field. The repulsion force in the vortex structure at the center of the sample that have resulted from its compaction, initiates the wave of the vortex density of the inverse direction of propagation. Upon reaching the surface, this wave is reflected from it. This results in the oscillations in the vortex system [6]. The limitation of the number of oscillations observed is caused by the existence of damping. One succeeds in observing the oscillation of the vortex density only owing to a strong compression of the vortex structure as a result of the giant avalanche-flux [6-11].

One important parameter characterizing a quantized vortices is the effective (intrinsic) mass, which can be associated with its motion [12-21]. Due to its importance, the concept of the vortex mass was discussed extensively over the years but remains a controversial issue. One point of view is that mass plays no role in the dynamics since an inertial term in the equation of motion of a vortex is always negligible next to the viscous drag force. Experimentally, the presence of an

inertial term is difficult to detect since at low temperatures vortices in superconductors are pinned, and if they move at all, their motion is dominated by viscosity. We are aware of only one such attempt, with inconclusive results⁸. It is quite clear that in order to check if an inertial term plays a significant role in vortex dynamics, one should realize conditions where the viscous drag force is not dominant. Our simulations will be performed in a region near the center of the sample where the currents vanish and this force is small [15].

In the present work, we study the the dynamics of the magnetic flux avalanches which takes account inertial properties of the vortex matter. It is shown that at under some conditions vortex instability can be observed during the flux avalanche process.

Bean [1] has proposed the critical state model which is successfully used to describe magnetic properties of type II superconductors. According to this model, the distribution of the magnetic flux density \vec{B} and the transport current density \vec{j} inside a superconductor is given by a solution of the equation

$$\text{rot} \vec{B} = \frac{4\pi}{c} \vec{j}. \quad (1)$$

When the penetrated magnetic flux changes with time, an electric field $\vec{E}(r, t)$ is generated inside the sample according to Faraday's law

$$\text{rot} \vec{E} = \frac{1}{c} \frac{d\vec{B}}{dt}. \quad (2)$$

In the flux flow regime the electric field $\vec{E}(r, t)$ induced by the moving vortices is related with the local current density $\vec{j}(r, t)$ by the nonlinear Ohm's law

$$\vec{E} = \bar{\nu} \vec{B}. \quad (3)$$

To obtain quantitative estimates, we use a classical equation of motion of a vortex, which it can derived by integrating over the microscopic degrees of freedom, leaving only macroscopic forces [21]. Thus, the equation of the vortex motion under the action of the Lorentz, pinning, and viscosity forces can be presented as

$$m \frac{dV}{dt} + \eta V + F_L + F_p = 0. \quad (4)$$

Here μ is the vortex mass per unit length, $\vec{F}_L = \frac{1}{c} \vec{j} \vec{\Phi}_0$ is the Lorentz force, $\vec{F}_p = \frac{1}{c} \vec{j}_c \vec{\Phi}_0$, $\eta = \frac{\Phi_0 H_{c2}}{c^2 \rho_n}$ is the flux flow viscosity coefficient, $\Phi_0 = \pi \hbar c / 2e$ is the magnetic flux quantum, H_{c2} is the upper critical field of superconductor, ρ_n is the normal state resistivity, j_c is the critical current density [4]. For simplicity we have neglected the Magnus force, assuming that it is much smaller than the viscous force (for example, for Nb see, [6]). In the absence of external currents and fields, the Lorentz force results from currents associated with vortices trapped in the sample.

In combining the relation (3) with Maxwell's equation (2), we obtain a nonlinear diffusion equation for the magnetic flux induction

$\vec{B}(r, t)$ in the following form

$$m \frac{dV}{dt} + \eta V = -\frac{1}{c} \Phi_0 (j - j_c), \quad (5)$$

$$\frac{d\vec{B}}{dt} = \nabla[\vec{v}\vec{B}]. \quad (6)$$

The temperature distribution in superconductor is governed by the heat conduction diffusion equation

$$\nu(T) \frac{dT}{dt} = \nabla[\kappa(T) \nabla T] + \vec{j} \vec{E}, \quad (7)$$

Here $\nu = \nu(T)$ and $\kappa = \kappa(T)$ are the specific heat and thermal conductivity, respectively. The above equations should be supplemented by a current-voltage characteristics of superconductors, which has the form

$$\vec{j} = j_c(T, \vec{B}, \vec{E}).$$

In order to obtain analytical results of a set Eqs. (5)-(7), we suggest that j_c is independent on magnetic field induction B and use the Bean critical state model $j_c = j_c(B_e, T)$, i.e., $j_c(T) = j_0 - a(T - T_0)$ [1]; where B_e is the external applied magnetic field induction, $a = j_0/(T_c - T_0)$, T_0 and T_c are the equilibrium and critical temperatures of the sample, respectively, j_0 is the equilibrium current density. For the sake of simplifying of the calculations, we perform our calculations on the assumption of negligibly small heating and assume that the temperature profile is a constant within the across sample and thermal conductivity κ and heat capacity ν are independent on the temperature profile [5].

We study the evolution of the thermal and electromagnetic penetration process in a simple geometry - superconducting semi-infinite sample $x \geq 0$. We assume that the external magnetic field induction B_e is parallel to the z-axis and the magnetic field sweep rate \dot{B}_e is constant. When the magnetic field with the flux density B_e is applied in the direction of the z-axis, the transport current $\delta j(x, t)$ and the electric field $\delta E(x, t)$ are induced inside the slab along the y-axis. For this geometry the spatial and temporal evolution of thermal and magnetic field perturbations

$$\begin{aligned} T &= T_0 + \Theta(x, t), \\ B &= B_e + b(x, t), \\ V &= V_0 + v(x, t) \end{aligned} \quad (8)$$

where $T_0(x)$, $B_e(x)$ and $V_0(x)$ are solutions to the unperturbed equations, which can be obtained within a quasi-stationary approximation; are described by the following system of differential equations [8, 11]

$$\frac{d\Theta}{dt} = 2v - \beta\Theta, \quad (9)$$

$$\mu \frac{dv}{dt} + v = -\frac{db}{dx} + \beta\Theta, \quad (10)$$

$$\frac{db}{dt} = \left(\frac{db}{dx} + b \right) + \left(\frac{dv}{dx} + v \right), \quad (11)$$

where we have introduced the dimensionless parameters

$$\mu = \frac{c\Phi_0}{4\pi\eta^2} \frac{B_e}{2L^2}, \quad \beta = \frac{4\pi}{c} \frac{j_c^2 L^2}{\nu(T_c - T_0)}.$$

$$b = \frac{B}{B_e} = \frac{c}{4\pi} \frac{B}{j_c L}, \quad \Theta = \frac{4\pi}{c} \frac{2\nu}{B_e^2}, \quad v = V \frac{t_0}{L}, \quad L = \frac{c}{4\pi} \frac{B_e}{j_c}.$$

variables

$$z = \frac{x}{L}, \quad \tau = \frac{t}{t_0} = \frac{c\Phi_0}{4\pi\eta} \frac{B_e}{2\mu_0 j_c L^2} t,$$

Here L is the magnetic field penetration depth, which is determined from equation (3)

$$B(x, t) = B_e + \frac{4\pi}{c} j_c (x - L), \quad (12)$$

with the appropriate boundary conditions

$$dB(0, t) = B_e, \quad B(L, t) = 0. \quad (13)$$

Assuming that the small thermal and magnetic perturbations has form $\Theta(x, t), b(x, t), v(x, t) \sim \exp[\gamma t]$, where γ is the eigenvalue of the problem to be determined, we obtained from the system Eqs. (9)-(11) the following dispersion relations to determine the eigenvalue problem

$$(\gamma + \beta) \frac{d^2 b}{dx^2} - [(\gamma + \beta)\mu - 2\beta] \frac{db}{dx} + [(\mu + 1)\gamma^2 + [(\mu - 1)\beta - \mu - 1]\gamma - (\mu - 1)\beta] b = 0 \quad (14)$$

The instability of the flux front is defined by the positive value of the rate increase $\text{Re } \gamma > 0$. The instability occurs at the condition

The instability of the flux front is defined by the positive value of the rate increase $\text{Re } \gamma > 0$.

An analysis of the dispersion relation shows that, the growth rate is positive $\text{Re } \gamma > 0$, if $\mu > \mu_c = 2$ and any small perturbations will grow with time. For the case when $\mu < \mu_c$, the growth rate is negative and the small perturbations will decay. At the critical value of $\mu = \mu_c$, the growth rate is zero $\gamma = 0$. For the specific case, where $\mu = 1$ the growth rate is determined by a stability parameter β . Thus, the stability criterion can be written as

$$\beta > 1.$$

For the case, where thermal effects is negligible ($\beta = 1$) we may obtain the following dispersion relation

$$\frac{d^2 b}{dx^2} - \mu \frac{db}{dx} + (\gamma - 1)(\mu + 1)b = 0. \quad (15)$$

Seeking for $b \sim \exp(ikx)$ in dispersion relation, the growth rate γ dependence can be obtained as a functions of wavenumber k .

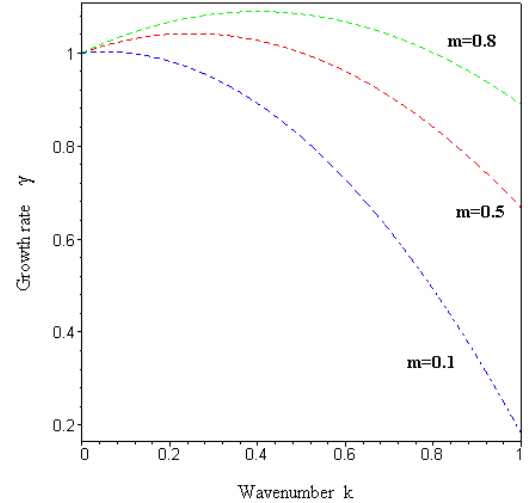


Fig.1. The dependence of the growth rate on the wavenumber for $\mu = 0.1, 0.5, 0.6$.

The stability of the system depends on the growth rate, γ , given in (15). We analyze the growth rate of small perturbations as a function of wavenumber k . When $k < k_c = \mu$ the growth rate is positive and any small perturbations will grow with time. For wave number $k > k_c$, the growth rate γ is negative. Consequently, the small perturbations always decay. It can be shown that, for wave number $k = k_c$ the growth rate is zero $\gamma = 0$. As the wave number approaches zero $k \rightarrow 0$ or infinity $k \rightarrow \infty$ the growth rate approaches $\gamma = 1$ and small perturbations grow with time. As the wave number approaches unity $k = 1$ the growth rate is determined by the value of μ

$$\gamma = \frac{2\mu}{\mu + 1}.$$

For $\mu = 0$ the growth rate is zero $\gamma = 0$. For $\mu = 1$ the growth rate is unity $\gamma = 1$. Since the growth rate is zero at the critical wave number and approaches to unity in the limit of zero wave number, there must exist a wave number in between that maximizes the growth rate. Fig. (1-4) shows the growing rate, γ , as a function of the wave number k , for various values mass μ . As the value of μ increases, the corresponding growth rate increases.

In the present work, we study the spatial and temporal evolution of small thermal and magnetic perturbation in type-II superconductor sample in the flux flux regime, assuming that an applied field parallel to the surface of the sample. On the basis of a linear analysis of a set of differential equations describing small perturbations of temperature and magnetic field we will found that under some conditions an instability may occur in the sample, which takes into account an inertial properties of the vortices mass.

References

1. C. P. Bean, Phys. Rev. Lett., 8, 250, 1962; Rev. Mod. Phys., 36, 31, 1964.
2. P. S. Swartz and S. P. Bean, J. Appl. Phys., 39, 4991, 1968.
3. S. L. Wipf, Cryogenics, 31, 936, 1961.
4. R. G. Mints, and A. L. Rakhmanov, Rev. Mod. Phys., 53, 551, 1981.
5. R. G. Mints and A. L. Rakhmanov, Instabilities in superconductors, Moscow, Nauka, 362, 1984.
6. V. V. Chabanenko, V. F. Rusakov, V. A. Yampol'skii, S. Piechota, A. Nabialek, S. V. Vasiliev, and H. Szymczak, arXiv: cond-mat/0106379v2, 2002.
7. S. Vasiliev, A. Nabialek, V. Chabanenko, V. Rusakov, S. Piechota, H. Szymczak, Acta Phys. Pol. A 109, 661, 2006.
8. A. Nabialek, S. Vasiliev, V. Chabanenko, V. Rusakov, S. Piechota, H. Szymczak, Acta Phys. Pol. A, 114, 2008.
9. S. Vasiliev, A. Nabialek, V. F. Rusakov, L. V. Belevtsov, V.V. Chabanenko and H. Szymczak, Acta Phys. Pol. A, 118, 2010.
10. V. Rusakov, S. Vasilieva, V.V. Chabanenko, A. Yurov, A. Nabialek, S. Piechotaa and H. Szymczak, Acta Phys. Pol. A, 109, 2006.
11. V. V. Chabanenko, V.F. Rusakov, A. Nabialek, S. Piechota, S. Vasiliev, H. Szymczak, Physica C, 369, 2002.
12. N. H. Zebouni, A. Venkataram, G. N. Rao, C. G. Grenier, J. M. Reynolds, Phys.Rev.Lett., 13, 606, 1964.
13. H. Suhl, Phys.Rev.Lett., 14, 226, 1965.
14. H. T. Coffey, Cryogenics, 7, 73, 1967.
15. N. V. Kopnin. Pis'ma v ZhETF 27, 417, 1978.
16. G. Baym, E. Chandler. J. Low Temp. Phys., 50, 57, 1983.
17. E. B. Sonin, V. B. Geshkenbein, A. van Otterlo, G. Blatter. Phys. Rev. B 57, 575, 1998.
18. M. J. Stephen, J. Bardin. Phys. Rev. Lett., 14 112, 1965.
19. G. E. Volovik, Pis'ma v ZhETF, 65, 201, 1997.
20. E. M. W. Coffey, Phys. Rev. B, 49, 9774, 1994.
21. J. I. Gittleman, B. Rosenblum. Journ. of Appl. Phys., 39, 2617, 1968.

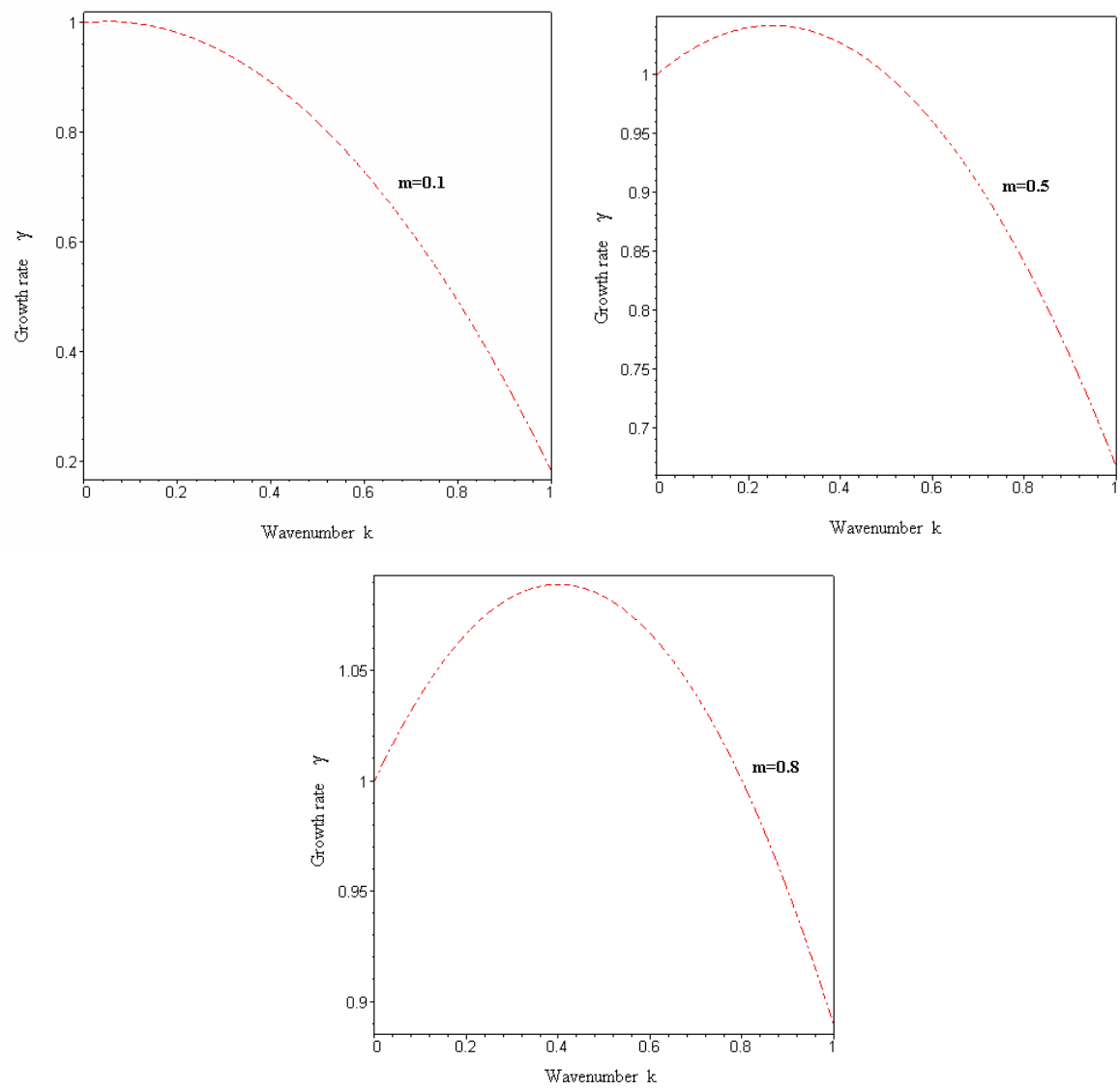


Fig.2-4. The dependence of the growth rate on the wavenumber for $m = 0.1, 0.5, 0.8$.